A Handbook on

Instrumentation Engineering

Contains well illustrated formulae & key theory concepts

for

GATE, PSUs

& OTHER COMPETITIVE EXAMS

MADE EASY PUBLICATIONS

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Director's Message

B. Singh (Ex. IES)

During the current age of international competition in Science and Technology, the Indian participation through skilled technical professionals have been challenging to the world. Constant efforts and desire to achieve top positions are still required.

I feel every candidate has ability to succeed but competitive environment and quality guidance is required to achieve high level goals. At MADE EASY, we help you to discover your hidden talent and success quotient to achieve your ultimate goals. In my opinion GATE & PSU's exams are tool to enter in to main stream of Nation serving. The real application of knowledge and talent starts, after you enter in to the working system. Here in MADE EASY you are also trained to become winner in your life and achieve job satisfaction.

MADE EASY alumni have shared their winning stories of success and expressed their gratitude towards quality guidance of MADE EASY. Our students have not only secured All India First Ranks in ESE, GATE and PSU entrance examinations but also secured top positions in their career profiles. Now, I invite you to become alumni of MADE EASY to explore and achieve ultimate goal of your life. I promise to provide you quality guidance with competitive environment which is far advanced and ahead than the reach of other institutions. You will get the guidance, support and inspiration that you need to reach the peak of your career.

I have true desire to serve Society and Nation by way of making easy path of the education for the people of India.

After a long experience of teaching in Instrumentation Engineering over the period of time MADE EASY team realised that there is a need of good *Handbook* which can provide the crux of Instrumentation Engineering in a concise form to the student to brush up the formulae and important concepts required for GATE, PSUs and other competitive examinations. This *handbook* contains all the formulae and important theoretical aspects of Instrumentation Engineering. It provides much needed revision aid and study guidance before examinations.

B. Singh (Ex. IES)
CMD, MADE EASY Group

A Handbook on

Instrumentation Engineering

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Instrumentation Engineering

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Engineering Mathematics

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Linear Algebra

1

MATRIX

Principal Diagonal: In a square matrix all elements a_{ij} for which $i = j$ are elements of principal diagonal.

Matrices

1. **Upper Triangular matrix:** A square matrix in which all the elements below the principle diagonal are zero.
2. **Lower Triangular Matrix:** A square matrix in which all the elements above the principle diagonal are zero.
3. **Diagonal Matrix:** A square matrix in which all the elements other than the elements of principle diagonal are zero.
4. **Scalar Matrix:** *A* diagonal matrix with all elements of principle diagonal being same.
5. **Idempotent Matrix:** 'A' is square matrix i.e. $A^2 = A$.
6. **Involutary Matrix:** 'A' is square matrix i.e. $A^2 = I$.
7. **Nilpotent Matrix:** 'A' is square matrix i.e. $A^m = 0$ where m is the least positive integer and m is also called as Index of class of Nilpotent matrix A.
8. **Transpose Matrix:** A^T is transpose matrix of matrix A. A^T can be obtained by switching the rows as columns and columns as rows of A.
9. **Symmetric Matrix:** 'A' is a square matrix i.e. $A^T = A$.
10. **Skew-Symmetric Matrix:** 'A' is a square matrix i.e. $A^T = -A$.
11. **Orthogonal Matrix:** 'A' is a orthogonal matrix i.e. $A^T = A^{-1}$ or $AA^T = I = A^TA$.
12. **Conjugate Matrix of A** ($\bar{A}$) or ($\sim A$) **:** 'A' is any matrix, by replacing the elements by corresponding conjugate complex numbers the matrix obtained is conjugate of 'A'.

Example:

$$A = \begin{bmatrix} 2+3i & 4+7i & 5 \\ 2i & 3 & 9-i \end{bmatrix} \Rightarrow \bar{A} = \begin{bmatrix} 2-3i & 4-7i & 5 \\ -2i & 3 & 9+i \end{bmatrix}$$

- The system of linear equation $AX = B$ has a solution (consistent) iff rank of A = Rank of $(A|B)$.
- The system $AX = B$ has
  (*i*) A unique solution iff Rank (A) = Rank $(A|B)$ = Number of variables.
  (*ii*) Infinitely many solutions ⇔ Rank (A) = Rank $(A|B)$ < number of variables.
  (*iii*) No solution if Rank $(A) \neq$ Rank $(A|B)$ i.e. Rank (A) < Rank $(A|B)$.
- The system $AX = 0$ has
  (*i*) Unique solution (zero solution or trivial solution) if Rank (A) = number of variables.
  (*ii*) Infinitely many number of solutions (non-trivial solutions) if Rank (A) < number of variables.
- If Rank $(A) = r$, and number of variables = n then, the number of linearly independent infinite solutions of $AX = 0$ is $(n - r)$.
- In the system of homogenous linear equation $AX = 0$
  (*i*) If A is singular then the system possesses non-trivial solution (i.e. infinite solution).
  (*ii*) If A is non-singular then the system possesses trivial (zero) solution (i.e. unique solution).
- Rank of a diagonal matrix = Number of non-zero elements in diagonal.
- If A and B are two matrices
  (*i*) $r(A + B) \leq r(A) + r(B)$
  (*ii*) $r(A - B) \geq r(A) - r(B)$
  (*iii*) $r(AB) \leq \min\{r(A), r(B)\}$
- If a matrix A has rank 'R', then A contains 'R' linearly independent vectors (row/column).
- The system of homogeneous linear equations such that number of unknowns (or variables) exceeds the number of equations necessarily possesses a non-zero solution.

Eigen Value

Let 'A' be a square matrix of order n and λ be a scalar then $|A - \lambda I| = 0$ is the characteristic equation of A. The roots of characteristic equation are called eigen values/lantent roots/Characteristic roots.

- The set of eigen values of matrix is called "spectrum of matrix".
- A matrix of order n will have n latent roots not necessarily distinct.

Differential Equations

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Differential Equation

Order of differential equation : It is the order of the highest derivative appearing in it.

Degree of Differential Equation : It is the degree of the highest derivative occurring in it, after expressing the equation free from radicals and fractions as far as derivatives are concerned.

Differential Equations of First Order First Degree

Equations of first order and first degree can be expressed in the form $y' = f(x, y)$. Following are the different ways of solving equations of first order and first degree:

1. Variable Separable : $f(x)dx + g(y)dy = 0$

   $\int f(x)dx + \int g(y)dy = c$ is the solution.

2. Homogenous Equation : $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$

   - To solve a homogeneous equation, substitute $y = Vx$

   $$\frac{dy}{dx} = V + x\frac{dV}{dx}$$

   - Separate the variable V and x and integrate.

Equations Reducible to Homogenous Equation

The differential equation: $\frac{dy}{dx} = \frac{ax + by + x'}{a'x + b'y + c'}$

This is non-homogeneous but can be converted to homogeneous equation.

Case I : If $\frac{a}{a'} \neq \frac{b}{b'}$

Substitude $x = X + h$, y $= Y + k$(h and K are constants)

Solve for h and k, we get

$$ah + bk + c = 0$$

$$a'h + b'k + c' = 0$$

$$\frac{dY}{dX} = \frac{aX + bY}{a'X + b'Y}$$

Case II : If $\frac{a}{a'} = \frac{b}{b'}$

$$\frac{a}{a'} = \frac{b}{b'} = \frac{1}{m} \text{ (say)}$$

$$\frac{dy}{dx} = \frac{(ax+by)+C}{m(ax+by)+C'}$$

Substitute $ax + by = t$, so that

$$\frac{dt}{dx} = \frac{b(t+C)}{mt+C^1} + a$$

Solve by variable separable method

3. Linear Equations : The standard form of a linear equation of first order

$\frac{dy}{dx} + P(x)y = Q(x)$, where P and Q are functions of x.

Second order linear equation $\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = R(x)$

It is commonly known as "Leibnitz's linear equations"

Integrating factor, $\text{I.F.} = e^{\int P dx}$

$$ye^{\int Pdx} = \int Q\cdot(I.F.) + dx + C \Rightarrow y(I.F.) = \int Q\cdot(I.F.)dx + C$$

Note:

- The degree of every linear differential equation is always one but if the degree of the differential equation is one, then it need not be linear.

For Ex. : $\frac{d^4y}{dx^4} + 3x^2\left(\frac{dy}{dx}\right)^3 + y^{2003} = 0$

Bernoulli's Equation

$$\frac{dy}{dx} + Py = Qy^n \quad \text{where } P \text{ \& } Q \text{ are functions of } x \text{ only}$$

Divide by y^n

$$y^{-n}\frac{dy}{dx} + Py^{1-n} = Q$$

Substituting, $y^{1-n} = z$, we get

$$\frac{dz}{dx} + (1-n)Pz = Q(1-n)]$$

This is a linear equation and can be solved easily.

Probability & Statistics

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MEAN, MEDIAN AND MODE

- $$\text{Mean } (\bar{X}) = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum f_i}$$

- $$\text{Median} = \frac{x_{n/2} + (x_{n/2} + 1)}{2};\ n \text{ is even}$$
$$= x_{n+1/2};\ n \text{ is odd}$$
$$= L_{med} + \left(\frac{N/2 - F}{f}\right) \times w$$

where L_{med} = lower limit of median class
N = Σf_i
F = Cumulative frequency upto the median class (cumulative frequency of the preceding class)
f = Frequency of the median class

Mode : Value of 'x' corresponding to maximum frequency.

$$L_{mode} + \frac{f_m - f_1}{(2f_m - f_1 - f_2)} \times h$$

where L_{mode} = lower limit of modal class
f_m = frequency of modal class
f_1 = preceding frequency of modal class
f_2 = Following frequency of modal class

Note: ....

- Mode = 3 Median – 2 Mean [for Asymmetric distribution]
- Mean = Mode = Median [for Symmetric distribution]

....

AXIOMS OF PROBABILITY

Let A and B be two events. Then

1. $P(\bar{A}) = 1 - P(A)$
2. $P(\phi) = 0$; ϕ is the empty set
3. $P(A - B) = P(A) - P(A \cap B)$
4. $P(A - B) = P(A \cap \bar{B})$
5. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
6. $P(A \cup B) = P(A) + P(B)$; mutually exclusive events.
7. $P(A \cap B) = P(A) \cdot P(B/A) = P(B) \cdot P(A/B)$
8. $P(A \cap B) = P(A) \cdot P(B)$; independent events.
9. $P(A \cap B) = \phi$; mutually exclusive events.
10. $P(S) = 1$; S is sample space.
11. $\max(0, P(A) + P(B) - 1) \le P(A \cap B) \le \min(P(A), P(B))$
12. $\max(P(A), P(B)) \le P(A \cup B) \le \min(1, P(A) + P(B))$
13. $\max(0, P(A_1) + P(A_2) \ldots + P(A_n) - (n - 1)) \le P(A_1 \cap A_2 \ldots \cap A_n)$
    $\le \min(P(A_1), P(A_2) \ldots, P(A_n))$
14. $\max(P(A_1), P(A_2) \ldots, P(A_n)) \le P(A_1 \cup A_2 \ldots \cup A_n)$
    $\le \min(1, P(A_1) + P(A_2) \ldots + P(A_n))$
15. $P(A | B) = \dfrac{P(A \quad B)}{P(B)}$
16. $P(A | B) = P(A)$; independent events.
17. $P(E_1 \cap E_2 \cap \ldots \cap E_n) = P(E_1) \cdot P(E_2) \ldots P(E_n)$; independent events.
18. Rule of total probability: $P(X) = \sum_{i=1}^{n} P(E_i)\, P(X | E_i)$
19. Baye's Theorem:

$$P(E_1 | X) = \frac{P(E_1) \cdot P(X|E_1)}{P(E_1) \cdot P(X|E_1) + P(E_2) \cdot P(X|E_2) + P(E_3) \cdot P(X|E_3)}$$

In general,

$$P(E_i | X) = \frac{P(E_i) \cdot P(X|E_i)}{\sum_{j=1}^{n} P(E_j) \cdot P(X|E_j)}$$

Numerical Methods 4

Solution of Algebraic and Transcendental Equation

Consider an equation $f(x) = 0$.

Bisection Method

This method finds the root between points '*a*' and '*b*'. If $f(x)$ is continuous between a and b and $f(a)$ and $f(b)$ are of opposite signs then their is a root between *a* and *b* (Intermediate Value Theorem). First approximation to the root is $x_1 = (a + b)/2$. If $f(x_1) = 0$, then x_1 is the root of $f(x) = 0$ otherwise root lies between a and x_1 or x_1 and *b*, Similarly, x_2 and x_3 ..... are determined.

- Simplest iterative method.
- Bisection method always converges, but often slowly.
- This method can't be used for finding the complex roots.
- Rate of convergence is linear.

Newton-Raphson Method (or Successive Substitution Method or Tangent Method)

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- This method is commonly used for its simplicity and greater speed.
- Here $f(x)$ is assumed to have continuous derivative $f'(x)$.
- This method fails if $f'(x) = 0$.
- It has second order of convergence or quadratic convergence i.e. the subsequent error at each step is proportional to the square of the error at previous step.
- Sensitive to starting value, i.e., the Newton's method converges provided the initial approximation is chosen sufficiently close to the root.
- Rate of convergence is quadratic.

Secant Method

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)$$

- Convergence is not guaranteed.
- It converges, convergence super.

Regula Falsi Method or (Method of False Position)

- Regula falsi method always converges.
- However, it converges slowly.
- If converges, order of convergence is between 1 and 2 (closer to 1).
- It is superior to bisection method.

Given $f(x) = 0$

Select x_0 and x_1 such that $f(x_0)\, f(x_1) < 0$ (*i.e.* opposite sign)

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0) = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

Check if $f(x_0)\, f(x_2) < 0$ $f(x_1)\, f(x_2) < 0$

Computer x_3 ....

which is an approximation to the root.

Solution of Linear System of Equations

Gauss Elimination Method

Here equations are converted into "upper triangular matrix" form, then solved by "back substitution" method.

Consider

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Step 1 : To eliminate x from second and third equation (we do this by subtracting suitable multiple of first equation from second and third equation)

$$a_1x + b_1y + c_1z = d_1 \quad \text{pivotal equation, } a_1 \text{ pivot point}$$
$$b_{2'}y + c_2z = d_2$$
$$b_{3'}y + c_{3'}z = d_3$$

Step 2 : Eliminate y from third equation

$$a_1x + b_1y + c_1z = d_{1'}$$
$$b_{2'}y + c_2z = d_{2'} \quad \text{pivotal equation, } b_{2'} \text{ pivot point}$$
$$c_{3^n}z = d_{3''}$$

Step 3 : The value of x, y and z can be found by back substitution.

Note: ....

- Number of operations : $N = \frac{n^3}{3} + n^2 - \frac{n}{3}$

....

Calculus

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LIMIT, CONTINUITY AND DIFFERENTIABILITY

Limit Existence at a Point

- **Left Limit (LHL):** $\lim_{x\to a^-} f(x) = \lim_{h\to 0} f(a-h)$

  Right Limit (RHL): $\lim_{x\to a^+} f(x) = \lim_{h\to 0} f(a+h)$

  $\lim_{x\to a} f(x)$ exists, iff both LHL and RHL exists.

- Indeterminate forms: 0/0, ∞/∞, $0\times\infty$, $\infty-\infty$, 0^0, 1^∞, 0^∞
- **L Hospital's Rule:** If $\lim_{x\to a}[f(x)/g(x)]$ is of the form either 0/0 or ∞/∞

  then $\lim_{x\to a}[f(x)/g(x)] = \lim_{x\to a}[f'(x)/g'(x)]$

Continuity

A function $f(x)$ is said to be continuous at $x = a$ if :

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = f(a)$$

- If $f(x)$ and $g(x)$ is continuous function then following are also continuous.

  (*i*) $f(x) + g(x)$

  (*ii*) $f(x)\cdot g(x)$

  (*iii*) $f(x) - g(x)$

  (*iv*) $f(x)/g(x)$ $[g(x)\neq 0]$
- Polynomial, Exponential, Sin and Cos functions are continuous everywhere.

Differentiability

A function $f(x)$ is said to be differentiable at x = a if it is continuous at $x = a$ and if left derivative = $f'(a)$ = right derivative

$$\text{i.e.}\quad \lim_{h\to 0^-}\frac{f(a)-f(a-h)}{h} = f'(a) = \lim_{h\to 0^+}\frac{f(a+h)-f(a)}{h}$$

- If $f(x)$ is differentiable in (a, b), then it is always continuous at (a, b), but converse need not be true.

MEAN VALUE THEOREMS

Rolle's Mean Value Theorem

If

(*i*) '$f(x)$' is continuous in $[a, b]$

(*ii*) '$f(x)$' is differentiable in (a, b)

(*iii*) $f(a) = f(b)$

then $\exists\ c \in (a, b)$ such that $f'(c) = 0$

Lagrange's Mean Value Theorem

If

(*i*) '$f(x)$' is continuous in $[a, b]$

(*ii*) '$f(x)$' is differentiable in (a, b)

then $\exists$ c $\in$ (a, b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a}$

Cauchy's Mean Value Theorem

Let $f(x)$ and $g(x)$ are two functions if

(*i*) $f(x)$ and $g(x)$ are continuous in $[a, b]$

(*ii*) $f(x)$ and $g(x)$ are differentiable in (a, b)

(*iii*) $g'(x) \neq 0\ \ \forall x \in (a, b)$

then $\exists\ c \in (a, b)$ such that $\dfrac{f'(c)}{g'(c)} = \dfrac{f(b) - f(a)}{g(b) - g(a)}$

Derivatives

- $\dfrac{d}{dx}(\sin x) = \cos x$
- $\dfrac{d}{dx}(\cos x) = -\sin x$
- $\dfrac{d}{dx}(\tan x) = \sec^2 x$
- $\dfrac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
- $\dfrac{d}{dx}(\sec x) = \sec x \cdot \tan x$
- $\dfrac{d}{dx}(\operatorname{cosec} x) = \operatorname{cosec} x \cot x$
- $\dfrac{d}{dx}(\sin^{-1} x) = \dfrac{1}{\sqrt{1 - x^2}}$
- $\dfrac{d}{dx}(\cos^{-1} x) = \dfrac{-1}{\sqrt{1 - x^2}}$
- $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1 + x^2}$
- $\dfrac{d}{dx}(\cot^{-1} x) = \dfrac{-1}{1 + x^2}$

Vector Calculus

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Vector Calculus

Scalar Point Function

If corresponding to each point P of region R, there is a corresponding scalar $\phi(P)$ is said to be a scalar point function for the region R.

$$\phi(P) = \phi(x, y, z)$$

Vector Point Function

If corresponding to each point P of region R, there corresponds a vector defined by $F(P)$, then F is called a vector point function for region R.

$$F(P) = F(x, y, z) = f_1(x,y,z)\hat{i} + f_2(x,y,z)\hat{j} + f_3(x,y,z)\hat{k}$$

Vector Differential Operator or Del Operator

$$\Delta = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$$

Directional Derivative

The directional derivative of f in a direction $\vec{N}$ is the resolved part of ∇f in direction $\vec{N}$.

$$\nabla f.\vec{N} = |\nabla f|\cos\alpha$$

where $\vec{N}$ is a unit vector in a particular direction.

Direction cosine: $l^2 + m^2 + n^2 = 1$,

where $l = \cos\alpha,\ m = \cos\beta,\ n = \cos\gamma$

Gradient

The vector function ∇f is defined as the gradent of the scalar point function $f(x, y, z)$ and written as grad f.

$$\text{Grad} f = \nabla f = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

- ∇f is vector function.
- If $f(x, y, z) = 0$ is any surface, then ∇f is a vector normal to the surface f and has a magnitude equal to rate of change of f along this normal.
- Direction derivative of $f(x, y, z)$ is maximum along ∇f and magnitude of this maximum is $|\nabla f|$.

Divergence

The divergence of a continuously differentiable vector point function F is denoted by div. F and is defined by the equation div. $F = \nabla . F$.

$$F = f\hat{i} + \phi\hat{j} + \psi\hat{k}$$

$$\text{div.} \quad F = \nabla . F = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(f\hat{i} + \Phi\hat{j} + \psi\hat{k}\right) = \frac{\partial f}{\partial x} + \frac{\partial \Phi}{\partial y} + \frac{\partial y}{\partial z}$$

- $\nabla \cdot f$ is scalar.
- $\nabla \cdot \nabla = \nabla^2$ is Laplacian operator.

Curl

The curl of a continuously differentiable vector point function F is denoted by curl F and is defined by the equation

$$\text{Curl } F = \nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f & \Phi & \psi \end{vmatrix}$$

$\nabla \times F$ is a vector function.

Solenoidal Vector Function

If $\nabla \cdot F = 0$, then A is called as solenoidal vector function.

Irrotational Vector Function

If $\nabla \cdot A = 0$, then A is said to be irrotational otherwise rotational.

Del Applied Twice to Point Functions

1. div. grad $f = \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$ ... this is Laplace equation.
2. curl grad $f = \nabla \times \nabla f = 0$
3. div. curl $F = \nabla \cdot \nabla \times F = 0$
4. curl curl $F = \nabla \times (\nabla \times F) = \nabla(\nabla \cdot F) - \nabla^2 \cdot F$
5. grad div $F = \nabla(\nabla \cdot F) = \nabla \times (\nabla \times F) + \nabla^2 \cdot F$

Vector Identities

- $\nabla(f + g) = \nabla f + \nabla g$
- $\nabla . (F + G) = \nabla . G + \nabla . G$
- $\nabla \times (F + G) = \nabla \times G + \nabla \times G$
- $\nabla(fg) = f\nabla g + g\nabla f$
- $\nabla . (fg) = \nabla f . G + f\nabla . G$
- $\nabla \times (fG) = \nabla f \times G + f\nabla \times G$
- $\nabla(F \cdot G) = F \times (\nabla \times G) + G \times (\nabla \times F)$
- $\nabla . (F \times G) = G \cdot (\nabla \times F) - F \cdot (\nabla \times G)$
- $\nabla \times (F \times G) = F(\nabla . G) - G(\nabla \times F)$